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CALCULATION OF THE CAVITATING FLOW AROUND A CIRCULAR CONE BY A SUBSONIC STREAM OF A COMPRESSIBLE FLUID†

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The problem of the flow around a circular cone according to the Ryabushinskii scheme by an axially symmetric subsonic compressible fluid is considered in a non-linear formulation. A numerical-analytical method is proposed for solving the problem, based on the use of the variables of a velocity hodograph, and is a development of the method proposed [1] for calculating the jet efflux from a funnel-shaped vessel. Results of a numerical investigation of the cavitating flow around a disc and cavitator with a conical channel by an axially symmetric subsonic stream of water are presented.

Problems of the axially symmetric cavitating flow of an incompressible fluid have been solved numerically in a non-linear formulation in many papers. Cavitators in the form of a sphere, a disc and a right circular cone were considered. The most important results which follow from an investigation of the cavitating flow around a disc and a cone by an incompressible fluid were obtained in [2, 3]. The results in [4, 5] are in good agreement with them.

Axially symmetric cavitating flows of a compressible fluid with a non-zero cavitation number Q have only been investigated previously in a linear formulation using thin-body gas dynamic methods [6–11].

1. Consider the axially symmetric subsonic flow around a circular cone by a compressible fluid according to the Ryabushinskii scheme. We assume that the fluid is ideal and weightless and that the flow is steady, irrotational and isentropic. In the plane of the cylindrical coordinates x, r, the domain occupied by the flow is bounded by the segments AB and HA of the axis of symmetry x, by the generatrices of the cones BC and HG which make an angle θ_0 with the axis of symmetry and by the arc CDG of the free surface (Fig. 1a). The origin of coordinates is chosen so that the plane $x=0$ serves as a plane of symmetry of the flow (the half-line DA belongs to this plane).

Let λ be the reduced velocity, M the Mach number, θ the angle of inclination of the velocity to the x-axis ($\theta=0$ on AB, HA and DA) and λ_a and λ_c the values of λ at an infinitely distant point A and in CDG, respectively ($\lambda_a<\lambda_c\leqslant 1$), $\tau=\lambda/\lambda_a$, $\tau_0=\lambda_c/\lambda_a$. In the (τ, θ)-plane, the parts of the flow domain lying to the left of the plane $x=0$ correspond to the rectangle

$$\Sigma_0=\{(\tau,\theta)\,|\,0<\tau<\tau_0, 0<\theta<\theta_0\}$$

(Fig. 1b; the segment BB_1 corresponds to the stagnation point B).

Introducing the velocity potential φ and the stream function ψ by means of the relationships

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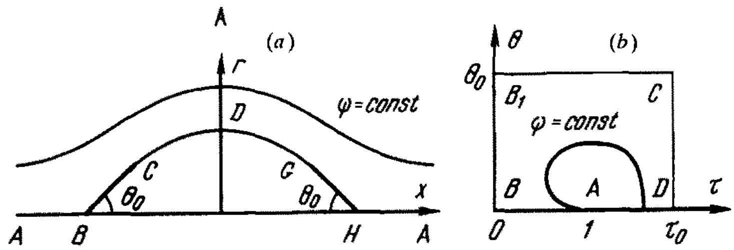

Fig. 1.

$$\tau\cos\theta = \varphi_x = (r\nu)^{-1}\psi_r, \quad \tau\sin\theta = \varphi_r = -(r\nu)^{-1}\psi_x, \quad \nu = \rho/\rho_0$$

and putting $z = x + ir$, we write

$$dz = \tau^{-1}[d\varphi + i(r\nu)^{-1}d\psi]e^{i\theta} \tag{1.1}$$

By treating τ and θ as the independent variables, we obtain from (1.1)

$$x_\kappa + ir_\kappa = \tau^{-1}[\varphi_\kappa + i(r\nu)^{-1}\psi_\kappa]e^{i\theta}, \quad \kappa = \tau, \theta \tag{1.2}$$

By cross differentiation of (1.2), we find

$$\varphi_\theta = \tau(r\nu)^{-1}\psi_\tau \tag{1.3}$$

$$\varphi_\tau = \left[\frac{\partial}{\partial\tau}\left(\frac{1}{r\nu}\right) - \frac{1}{r\nu\tau}\right]\psi_\theta - \frac{1}{\nu}\frac{\partial}{\partial\theta}\left(\frac{1}{r}\right)\psi_\tau \tag{1.4}$$

Using Euler's equation $\rho V dV + dp = 0$ and the relationship $dp/d\rho = a^2$ (V is the modulus of the velocity and a is the velocity of sound), we reduce (1.4) to the form

$$\varphi_\tau = (r\nu\tau)^{-1}(M^2 - 1)\psi_\theta + (r\nu\tau\sin\theta)^{-1}F \tag{1.5}$$

$$F = \tau r^{-1}(\psi_\tau r_\theta - \psi_\theta r_\tau)\sin\theta \tag{1.6}$$

Using (1.3) and (1.5), we obtain from (1.2)

$$\begin{aligned} rx_\tau &= [(M^2 - 1)\psi_\theta\cos\theta + F\operatorname{ctg}\theta - \tau\psi_\tau\sin\theta](\nu\tau^2)^{-1} \\ rx_\theta &= [\tau\psi_\tau\cos\theta - \psi_\theta\sin\theta](\nu\tau)^{-1} \end{aligned} \tag{1.7}$$

$$y_\tau = G = [(M^2 - 1)\psi_\theta\sin\theta + F + \tau\psi_\tau\cos\theta](\nu\tau^2)^{-1}$$

$$y_\theta = H = [\tau\psi_\tau\sin\theta + \psi_\theta\cos\theta](\nu\tau)^{-1}, \quad y = r^2/2 \tag{1.8}$$

From (1.6) and (1.8), we find

$$F = \sin^2\theta[\tau^2\psi_\tau^2 + (1 - M^2)\psi_\theta^2](2Y + \psi_\theta\sin\theta)^{-1}, \quad Y = \nu\tau y \tag{1.9}$$

Eliminating φ from (1.3) and (1.5), we arrive at the equation

$$\begin{aligned} &L(\psi) = N(\psi, Y), \quad N(\psi, Y) = \frac{1}{\sin\theta}\frac{\partial F}{\partial\theta} \\ &L(\psi) = (1 - M^2)\psi_{\theta\theta} + \tau^2\psi_{\tau\tau} + (1 + M^2)\tau\psi_\tau \end{aligned} \tag{1.10}$$

In deriving formulae (1.7)–(1.10) we have used the assumptions regarding the steady, barotropic, potential and adiabatic nature of the flow but no actual relationship between the gas dynamic parameters. Relationships (1.7)–(1.10) (which were first obtained in [1]) are therefore applicable to the investigation of flows of compressible liquids and gases with different equations of state.

The following conditions must be satisfied on the boundary of the domain Σ_0

$$\psi=0 \text{ on } ABB_1CD, \quad \psi_\theta=0 \text{ on } AD, \quad y=0 \text{ on } ABB_1 \tag{1.11}$$

Using (1.8), (1.9) and (1.11), Y can be expressed in terms of ψ

$$Y=Y(\psi)=\psi\cos\theta+\int_0^\theta(\tau\psi_\tau+\psi)\sin\theta\, d\theta+\Omega(\psi) \tag{1.12}$$

$$\Omega(\psi)=\nu\tau\int_1^\tau(\nu\tau^2)^{-1}(1-M^2)\psi|_{\theta=0}\, d\tau$$

Hence, the problem reduces to determining the function $\psi(\tau, \theta)$, which satisfies relationships (1.10), (1.9) and (1.12) in the domain Σ_0, with the boundary conditions (1.11) and

$$\psi>0, \quad (\tau,\theta)\in\Sigma_0 \tag{1.13}$$

On CD, we have $\psi_\theta=0$ and the following formulae for the curvature K of the arc CD is therefore obtained from (1.7) and (1.8)

$$K=\left.\frac{x_\theta r_{\theta\theta}-r_\theta x_{\theta\theta}}{(x_\theta^2+r_\theta^2)^{3/2}}\right|_{\tau=\tau_0}=-\nu r\psi_\tau^{-1}\Big|_{\tau=\tau_0} \tag{1.14}$$

Since $\psi_\tau=0$ on B_1C, it follows from (1.4) that $K\to\infty$ on approaching the point of contact of the free surface with the surface of the cone (this assertion has been proved in [12] in the case of an incompressible fluid).

2. Let us represent the solution of the problem in the form $\psi=\psi^0+\chi$, where ψ^0 is the singular part of ψ which describes the behaviour of the stream function in the neighbourhood of point A and determines the topology of the flow. The method of asymptotic expansions is used in the search for ψ^0.

Let us put

$$\omega=\operatorname{arctg}\frac{\theta}{\alpha\zeta}, \quad \zeta=\tau-1, \quad \alpha=(1-M_a^2)^{1/2} \tag{2.1}$$

Here, M_a is the value of M when $\lambda=\lambda_a$ and $\omega\in[0, \pi]$ when $(\tau, \theta)\in\Sigma_0$.

We shall seek the asymptotic expansion of the function $\psi^0=\psi^0(\theta, \omega)$ with respect to θ, representing the principal term ψ^{01} in the form $\psi^{01}=\theta^{-n}f_1(\omega)$ ($n=\text{const}$, $n>0$). Here

$$\begin{aligned}
&\psi_\tau^{01}=-\alpha\theta^{-n-1}\sin^2\omega f_1', \quad \psi_\theta^{01}=\theta^{-n-1}(-nf_1+\tfrac{1}{2}\sin 2\omega f_1')\\
&\psi_{\tau\tau}^{01}=\alpha^2\theta^{-n-2}(2\sin^3\omega\cos\omega f_1'+\sin^4\omega f_1'')\\
&\psi_{\theta\theta}^{01}=\theta^{-n-2}[n(n+1)f_1-(n+\sin^2\omega)\sin 2\omega f_1'+\tfrac{1}{4}\sin^2 2\omega f_1'']
\end{aligned} \tag{2.2}$$

In accordance with (1.11) and (1.13), we subject ψ^{01} to the conditions

$$\psi_\theta^{01}=0,\ \omega=0;\quad \psi^{01}=0,\ \omega=\pi;\quad \psi^{01}>0,\ 0\leqslant\omega<\pi \tag{2.3}$$

When account is taken of (2.1) and (2.2), we obtain from the first equality of (2.3) that

$$\omega^{-n-1}(-nf_1+\sin\omega f_1')\to 0,\ f_1=O(\omega^n),\ \psi^{01}=O(\zeta^{-n})\quad\text{as}\quad \omega\to 0 \tag{2.4}$$

In expression (1.12), we put $\psi=\psi^{01}$ and change from the variables τ, θ to the variables θ, ω (since $\tau-1=\alpha^{-1}\theta\operatorname{ctg}\omega$, any analytic function τ can be represented in the form of a series in powers of θ with coefficients which depend on ω). When account is taken of (2.1), (2.2) and (2.4), we shall have

$$\Omega(\psi^{01})=O(\zeta^{-n+1})=O(\theta^{-n+1}),\quad \int_0^\theta \tau\psi_\tau^{01}\sin\theta\, d\theta=O(\theta^{-n+1})$$

$$\int_0^\theta \psi^{01}\sin\theta\, d\theta=O(\theta^{-n+2}),\quad Y(\psi^{01})=\theta^{-n}f_1(\omega)+O(\theta^{-n+1})$$

In relation (1.10), we put $\psi=\psi^{01}$ and change to the variables θ, ω. On equating the principal terms in the expansions of the left- and right-hand sides of (1.10) in powers of θ (terms of the order of θ^{-n-2}), we obtain a differential equation in $f_1(\omega)$ from which, after the substitution $f_1(\omega)=\sin^n\omega\varphi_1(\omega)$ we obtain the equation

$$\begin{aligned}&[4+(n^2-4n)\sin^2\omega]\varphi_1^2\varphi_1''+[(4+2n-n^2)\sin^2\omega-4]\varphi_1\varphi_1'^2+\tfrac{1}{2}\sin 2\omega(\varphi_1'^3+n^2\varphi_1^2\varphi_1')+\\&+(4n^2-2n^3)\sin^2\omega\varphi_1^3=0\end{aligned} \tag{2.5}$$

The conditions

$$\varphi_1'(0)=0;\ \varphi_1(\pi)=0;\ \varphi_1(\omega)>0,\ 0\leqslant\omega<\pi \tag{2.6}$$

are obtained from (2.3) for $\varphi_1(\omega)$.

Analysis shows that a unique solution (apart from a constant factor) of problem (2.5), (2.6) exists and that it holds when $n=2/3$. The required function φ_1 is found by numerical integration of (2.5) from the point $\omega=\pi-\varepsilon$ $(0<\varepsilon\ll 1)$ to $\omega=0$ taking account of the fact that the expansion

$$\varphi_1(\pi-t)=t^2-\frac{1}{3}t^4+\frac{28}{405}t^6-\frac{5129}{612360}t^8+\ldots \tag{2.7}$$

holds in the neighbourhood of the point $\omega=\pi$.

In the neighbourhood of the point $\omega=0$

$$\varphi_1(\omega)=b\left(1-\frac{2}{81}\omega^4-\frac{4}{2187}\omega^6+\frac{91}{229635}\omega^8+\ldots\right) \tag{2.8}$$

where $b=1.4174112$ (it is found by numerical integration). The function $\varphi_1(\omega)$ decreases monotonically from the value b to zero in the interval $0\leqslant\omega\leqslant\pi$ while $\varphi_1'(\omega)$ vanishes at the ends of the interval and has a single minimum. A plot of the function $\varphi_1(\omega)$ is shown in Fig. 2, curve 1.

Hence, $\psi^{01}=\theta^{-2/3}\sin^{2/3}\omega\varphi_1(\omega)=\mu^{-2/3}\varphi_1(\omega)$, where $\mu=(\sigma^2+\theta^2)^{1/2}$, $\sigma=\alpha(\tau-1)$. The quantities η and ω serve as polar coordinates in the affinely transformed plane of the hodograph σ, θ and all curves $\psi^{01}=\text{const}$ are similar. On approaching the singular point from all directions $\omega=\text{const}$, apart from $\omega=\pi$, ψ^{01}, tends to infinity as $\mu^{-2/3}$ (when $\omega=\pi\psi^{01}=0$).

A discrepancy of the order of $\theta^{-2/3}$ is obtained as the result of the substitution of ψ^{01} into Eq. (1.10). It is natural to seek the term following ψ^{01} in the expansion of $\psi^0(\theta,\omega)$ with respect to θ in the form $\psi^{02}=\theta^{1/3}f_2(\omega)$ while requiring that a discrepancy of the order of $\theta^{-2/3}$

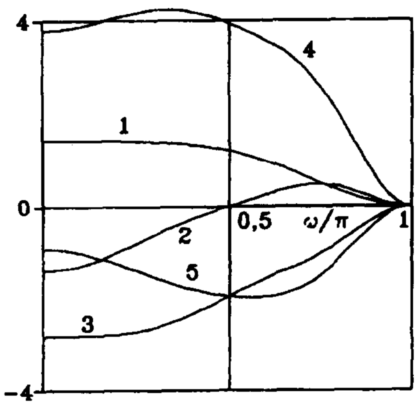

Fig. 2.

should be obtained as the result of substituting $\psi^{01}+\psi^{02}$ into (1.10).

By requiring that $\psi^{02}_\theta=0$ when $\omega=0$ and $\psi^{02}=0$ when $\omega=\pi$, we obtain the boundary conditions for $f_2(\omega)$

$$\lim_{\omega\to 0}\omega^{-2/3}\left(\frac{1}{3}f_2+\frac{1}{2}\sin 2\omega f_2'\right)=0,\quad \lim_{\omega\to\pi}(\pi-\omega)^{1/3}f_2=0 \tag{2.9}$$

It follows from (2.9), in particular, that $f_2=O(\omega^{-1/3})$, $\psi^{02}=O(\xi^{1/3})$ when $\omega\to 0$ and, hence, $\Omega(\psi^{02})=O(\xi^{4/3})=O(\theta^{4/3})$. When account is taken of the last relationship of (2.2), we find

$$\begin{aligned}
&Y(\psi^{01}+\psi^{02})=\theta^{-2/3}f_1(\omega)+\theta^{1/3}f_2(\omega)+\alpha\theta^{1/3}F_1(\omega)+O(\theta^{4/3})\\
&F_1(\omega)=3b(\operatorname{ctg}\omega)^{1/3}+F_0(\omega),\quad \omega\in[0,\pi/2]\\
&F_1(\omega)=F_0(\omega),\quad \omega\in[\pi/2,\pi]\\
&F_0(\omega)=-(\operatorname{ctg}^2\omega)^{1/6}\int_0^\omega(\operatorname{tg}^2\omega)^{2/3}f_1'(\omega)d\omega,\quad \omega\in[0,\pi/2)\\
&F_0(\omega)=(\operatorname{ctg}^2\omega)^{1/6}\int_\pi^\omega(\operatorname{tg}^2\omega)^{2/3}f_1'(\omega)d\omega,\quad \omega\in(\pi/2,\pi]\\
&F_0(\pi/2)=-3f_1'(\pi/2)
\end{aligned} \tag{2.10}$$

From (2.10), taking account of (2.7) and (2.8), we obtain the expansions

$$\begin{aligned}
&F_1(\omega)=3b\omega^{-1/3}\left(1-\frac{2}{9}\omega^2-\frac{1}{135}\omega^4+\ldots\right)\\
&F_1(\pi-t)=\frac{2}{3}t^{11/3}\left(1-\frac{1}{3}t^2+\frac{2}{45}t^4+\ldots\right)
\end{aligned} \tag{2.11}$$

It can be shown that the function $F_1(\omega)$ satisfies the equation

$$F_1'(\omega)+\frac{2}{3}\csc 2\omega F_1(\omega)=-\operatorname{tg}\omega f_1'(\omega) \tag{2.12}$$

and is analytic in the interval $(0,\pi)$ and can be represented in the neighbourhood of the point $\omega=\pi/2$ in the form

$$F_1\left(\frac{\pi}{2}+t\right)=\sqrt{b}\left(\sqrt{2}-\frac{13}{6}t+\frac{49\sqrt{2}}{144}t^2+\frac{59}{243}t^3+\frac{12829\sqrt{2}}{124416}t^4+\dots\right)$$

The quantities $F_1(\omega)$, $F_1'(\omega)$ outside of the neighbourhood of the point $\omega=\pi/2$ can be found using formulae (2.10) or by numerical integration of (2.12) using (2.11).

Let us put $\psi=\psi^{01}+\psi^{02}$ in (1.10) and change to the variables θ, ω. Equating terms of the order of $\theta^{-5/3}$ in the expansions of the left-hand and right-hand sides of (1.1) in powers of θ, we obtain

$$Pf_2''+Qf_2'+Rf_2=S,\quad P=4\sin^2\omega\left(1-\frac{5}{9}\sin^2\omega\right)f_1^2 \tag{2.13}$$

$$Q=\frac{14}{3}\sin 2\omega\left(1-\frac{10}{21}\sin^2\omega\right)f_1^2-12\sin^2\omega\left(1-\frac{25}{27}\sin^2\omega\right)f_1f_1'+3\sin^3\omega\cos\omega f_1'^2$$

$$R=8\sin^2\omega\left(1-\frac{2}{9}\sin^2\omega\right)f_1f_1''-\frac{14}{3}\sin 2\omega\left(1+\frac{8}{21}\sin^2\omega\right)f_1f_1'+\frac{76}{9}f_1^2-$$

$$-3\sin^2\omega\left(1-\frac{62}{27}\sin^2\omega\right)f_1'^2$$

$$S=\alpha F_1\left[-\frac{200}{27}f_1^2+\frac{28}{9}\sin 2\omega\left(1+\frac{6}{7}\sin^2\omega\right)f_1f_1'+\frac{10}{3}\sin^2\omega\cos 2\omega f_1'^2-\right.$$

$$\left.-8\sin^2\omega\left(1-\frac{1}{3}\sin^2\omega\right)f_1f_1''\right]-\alpha F_1'\sin 2\omega\left[\frac{4}{9}f_1^2-\frac{2}{3}\sin 2\omega f_1f_1'+\sin^2\omega f_1'^2\right]+$$

$$+a_1\left[\sin\omega\cos^3\omega(6f_1f_1'^2-4f_1^2f_1'')-\frac{64}{27}\operatorname{ctg}\omega f_1^3-\frac{4}{3}\cos^2\omega(1-6\sin^2\omega)f_1^2f_1'-\right.$$

$$\left.-\sin^2\omega\cos^4\omega f_1'^3\right]+(c_1-b_1)\sin^4\omega\cos^2\omega f_1'^3+\left(\frac{16}{9}c_1-\frac{32}{9}b_1\cos^2\omega\right)\sin^2\omega f_1^2f_1'+$$

$$+\left(\frac{8}{3}c_1-\frac{8}{9}b_1\right)\sin^3\omega\cos\omega f_1f_1'^2-\frac{16}{9}b_1\sin^3\omega\cos\omega f_1^2f_1''$$

$$a_1=\frac{\lambda_a}{(1-M_a^2)^{3/2}}\frac{d}{d\lambda}(1-M^2)\Big|_{\lambda=\lambda_a}$$

$$b_1=2(1-M_a^2)^{-1/2},\quad c_1=(1+M_a^2)(1-M_a^2)^{-1/2}$$

Using (2.7), (2.8) and (2.11), it can be shown that, when $\omega\to 0$ and $\omega=\pi-t$, $t\to 0$ the expansions

$$P(\omega)=b^2\omega^{10/3}\left(4-\frac{40}{9}\omega^2+\dots\right),\quad P(\pi-t)=t^{22/3}\left(4-\frac{64}{9}t^2+\dots\right)$$

$$Q(\omega)=b^2\omega^{7/3}\left(\frac{8}{3}-\frac{20}{27}\omega^2+\dots\right),\quad Q(\pi-t)=t^{19/3}\left(\frac{4}{3}-\frac{160}{27}t^2+\dots\right)$$

$$R(\omega)=b^2\omega^{4/3}\left(-\frac{8}{9}+\frac{428}{81}\omega^2+\dots\right),\quad R(\pi-t)=t^{16/3}\left(-\frac{20}{9}+\frac{1856}{81}t^2+\dots\right) \tag{2.14}$$

$$S(\omega)=b^3\omega^3\sum_{n=1}^{\infty}\sigma_n\omega^{2n-2},\quad S(\pi-t)=t^9\sum_{n=1}^{\infty}\nu_nt^{2n-2}$$

$$\sigma_1=\frac{8}{3}(a_1+c_1-b_1-2\alpha),\quad \nu_1=\frac{128}{3}\left(b_1-c_1-a_1-\frac{1}{2}\alpha\right)$$

hold.

It follows from (2.9) and (2.13) that the following boundary-value problem holds in the case of the function $\varphi_2(\omega)=\sin^{1/3}\omega f_2(\omega)$

$$P_1\varphi_2''+Q_1\varphi_2'+R_1\varphi_2=S_1,\quad P_1=P,\quad Q_1=-\frac{2}{3}\operatorname{ctg}\omega P+Q$$
$$R_1=\left(\frac{4}{9}\operatorname{ctg}^2\omega+\frac{1}{3}\right)P-\frac{1}{3}\operatorname{ctg}\omega Q+R,\quad S_1=\sin^{1/3}\omega S \tag{2.15}$$

$$\varphi_2'(0)=0,\quad \varphi_2(\pi)=0 \tag{2.16}$$

By substituting (2.14) into (2.15), it can be shown that $\varphi_2(\omega)$ can be expanded in the form

$$\varphi_2(\omega)=\sum_{n=1}^{\infty}p_n\omega^{n-1},\quad \varphi_2(\pi-t)=\sum_{n=1}^{\infty}q_nt^{2n} \tag{2.17}$$

and, moreover, the relationships

$$p_{2j}=\alpha_jp_2,\quad p_{2j+1}=\beta_j+\gamma_jp_1,\quad q_j=\mu_j+\kappa_jq_1,\quad j=1,2,\ldots,$$

hold in which the coefficients α_j, β_j, γ_j, μ_j, κ_j are expressed in terms of the coefficients of the expansions (2.14). In particular

$$p_3=\frac{1}{8}b\sigma_1-\frac{1}{2}p_1,\quad q_2=\frac{1}{32}\nu_1-\frac{5}{6}q_1 \tag{2.18}$$

The second of the conditions (2.16) is satisfied in the case of the expansions (2.17) and the first is equivalent to the condition $p_2=0$. The problem therefore reduces to searching for a q_1 for which, on integrating (2.15) numerically from the point $\omega=\pi-\varepsilon$ $(0<\varepsilon\ll 1)$ using (2.17) and (2.18) to determine the values of $\varphi_2(\pi-\varepsilon)$, $\varphi_2'(\pi-\varepsilon)$, a function $\varphi_2(\omega)$ is obtained which satisfies the condition $p_2=\varphi_2'(0)=0$.

The right-hand side of Eq. (2.15) can be represented in the form

$$S_1=a_1S_{1a}+k_1S_{1k}+m_1S_{1m}+n_1S_{1n} \tag{2.19}$$

where $k_1=(1-M_a^2)^{-1/2}$, $m_1=k_1M_a^2$, $n_1=k_1M_a^4$, S_{1a}, S_{1k}, S_{1m}, S_{1n} are unknown functions of ω which are independent of a_1 and M_a. It is therefore convenient to use the representation

$$\varphi_2=a_1\varphi_{2a}+k_1\varphi_{2k}+m_1\varphi_{2m}+n_1\varphi_{2n} \tag{2.20}$$

and to determine each of the functions φ_{2a}, φ_{2k}, φ_{2m}, φ_{2n} which depend solely on ω from the solution of the corresponding boundary-value problem which is obtained when (2.19) and (2.20) are substituted into (2.15) and (2.16).

Analysis shows that the expansions

$$\varphi_{2a}(\omega)=b\left(-1+\frac{5}{6}\omega^2+\ldots\right),\quad \varphi_{2a}(\pi-t)=2t^2-3t^4+\ldots$$

$$\varphi_{2k}(\omega)=b(-2+O(\omega^3)\ldots),\quad \varphi_{2k}(\pi-t)=-3t^2+\frac{19}{6}t^4+\ldots$$

$$\varphi_{2m}(\omega)=b\left(\frac{8}{3}+\frac{2}{3}\omega^2+\ldots\right),\quad \varphi_{2m}(\pi-t)=\frac{22}{3}t^2-\frac{67}{9}t^4+\ldots$$

$$\varphi_{2n}(\omega)=b\left(-\frac{2}{3}-\frac{2}{3}\omega^2+\ldots\right),\quad \varphi_{2n}(\pi-t)=-\frac{13}{3}t^2+\frac{77}{18}t^4+\ldots$$

hold when $\omega \to 0$ and $\omega = \pi - t$, $t \to 0$.

The dependences of φ_{2a}, φ_{2k}, φ_{2m}, φ_{2n} on ω are shown in Fig. 2 by curves 2–5.

Let $\varphi_2(\omega)$ be the solution of (2.15), (2.16). Then $\psi^{02} = \theta^{1/3} \sin^{-1/3} \omega \varphi_2(\omega) = \mu^{1/3} \varphi_2(\omega)$. Hence, the families $\psi^{02} = c > 0$ and $\psi^{02} = c < 0$ (c = const) in the (σ, θ)-plane consist of similar curves and they tend to zero as $\mu^{1/3}$ on approaching the singular point from any direction ω = const ψ^{02}.

3. Let us assume the function ψ^0 has been found with sufficiently high accuracy.

The function $\chi = \psi - \psi^0$ must serve as a solution of the boundary-value problem

$$L(\chi) = N(\psi^0 + \chi, Y(\psi^0 + \chi)) - \dot{L}(\psi^0), \quad (\tau, \theta) \in \Sigma_0 \tag{3.1}$$

$$\chi = -\psi^0 \quad \text{in} \quad ABB_1CD, \quad \chi_\theta = 0 \quad \text{in} \quad AD \tag{3.2}$$

There is practical interest in the cases when the magnitude of λ_c is close to λ_a (the magnitude of τ_0 is close to unity) and the gradients of the quantities which are determined in the interval AD and the part of the domain Σ_o adjoining it are large. When the problem is solved numerically, this leads to the need for a transformation of the independent variables.

We shall use the following transformations which convert Σ_0 into $\Sigma_1 = \{(\xi, \eta) | 0 < \xi < 1, 0 < \eta < 1\}$. When

$$\tau_0 < 2, \quad \frac{1}{2} \theta_0 \{\alpha \tau_0 (\tau_0 - 1)[\ln 2 - \ln(\tau_0 - 1)]\}^{-1} \geqslant 1 \tag{3.3}$$

we put [13, Section 5.6]

$$\xi = f(\tau) = F(\beta_1, \tau / \tau_0), \quad \eta = g(\theta) = 1 - F(\beta_2, 1 - \theta / \theta_0)$$
$$F(x, y) = \ln[(x+y)/(x-y)]\{\ln[(x+1)/(x-1)]\}^{-1}$$

and determine the parameters β_1, β_2 ($1 < \beta_1$, $\beta_2 < \infty$) from the conditions

$$f(1) = 0{,}5, \qquad f'(1) = \alpha g'(0) \tag{3.4}$$

(conditions (3.3) guarantee the unique solvability of Eqs (3.4) with respect to β_1 and β_2). When only the first of the inequalities (3.3) is satisfied, we put $\xi = f(\tau)$, $f(1) = 0.5$, $\eta = \theta / \theta_0$. When $\tau_0 \geqslant 2$ let $\xi = \tau / \tau_0$, $\eta = \theta / \theta_0$.

Equation (3.1) is written in the form

$$\Gamma(\chi) = N(\psi^0 + \chi, \; Y(\psi^0 + \chi)) - L(\psi^0) \tag{3.5}$$
$$\Gamma(\chi) = A\chi_{\eta\eta} + B\chi_{\xi\xi} + C\chi_\xi + D\chi_\eta$$
$$A = (1 - M^2)\eta_\theta^2, \quad B = \tau^2 \xi_\tau^2, \quad D = (1 - M^2)\eta_{\theta\theta}, \quad C = \tau^2 \xi_{\tau\tau} + (1 + M^2)\tau\xi_\tau$$

We introduce a uniform rectangular grid in the (ξ, η)-plane with step sizes $\Delta\xi$ with respect to ξ and $\Delta\eta$ with respect to η and with grid points at the points (ξ_i, η_j), where $\xi_1 = i\Delta\xi$, $\eta_j = i\Delta\eta$, $i = 0, 1, \ldots, I$, $j = 0, 1, \ldots, J$, $\Delta\xi I = \Delta\eta J = 1$. Let ξ_a be the value of ξ corresponding to $\tau = 1$ and let m be an index for which $\xi_{m-1} \leqslant \xi_a$, $\xi_m > \xi_a$. We shall denote quantities calculated at a point (ξ_i, η_j) with the subscripts i, j and those calculated on the line $\xi = \xi_i$ by the subscript i.

The determination of χ reduces to solving an iterative sequence of linear boundary-value difference problems: $\chi^{(n+1)}(n+1)$ (the approximation of the required function) is found using the scheme

$$\chi^{(n+1)} = (1 - w)\chi^{(n)} + w\chi^{(n+1/2)}, \quad 0 < w \leqslant 1, \quad n = 0, 1, \ldots$$

and the solution of the difference problem

$$
\begin{aligned}
&\Lambda(\chi)_{i,j} = T_{i,j}^{(n)}, \quad i=1,2,\ldots,I-1, \quad j=1,2,\ldots,J-1 \\
&\Lambda(\chi)_{i,j} = A_{i,j}(\chi_{i,j+1}-2\chi_{i,j}+\chi_{i,j-1})/\Delta\eta^2 + D_{i,j}(\chi_{i,j+1}-\chi_{i,j-1})/2\Delta\eta + \\
&+B_i(\chi_{i+1,j}-2\chi_{i,j}+\chi_{i-1,j})/\Delta\xi^2 + C_i(\chi_{i+1,j}-\chi_{i-1,j})/2\Delta\xi \\
&T^{(n)} = N(\psi^0+\chi^{(n)},\ Y^{(n)}) - L(\psi^0) \\
&\Lambda(\chi)_{i,0} = U_{i,0}^{(n)}, \quad \chi_{i,-1}=\chi_{i,1}, \quad i=m,m+1,\ldots,I-1 \\
&U^{(n)} = \tau^2(\psi_\tau^0+\chi_\tau^{(n)})^2/Y^{(n)} - L(\psi^0) \\
&\chi_{0,j} = -\psi_{0,j}^0, \quad \chi_{I,j} = -\psi_{I,j}^0, \quad j=0,1,\ldots,J \\
&\chi_{i,J} = -\psi_{i,J}^0, \quad i=0,1,\ldots,I; \quad \chi_{i,0}=0, \quad i=0,1,\ldots,m-1
\end{aligned}
\tag{3.6}
$$

is accepted for $\chi^{(n+1/2)}$.

The method of successive upper relaxation is used in the implementation of the difference scheme (3.6). The partial derivatives of $\chi^{(n)}$, which occur in the expressions $T^{(n)}$, $U^{(n)}$ are calculated on the basis of a spline approximation of the grid values of $\chi^{(n)}$.

As a rule, domains of negative values of the quantities $\psi^{(n)} = \psi^0 + \chi^{(n)}$, $Y(\psi^{(n)}) + \psi_\theta^{(n)}\sin\theta$, arise outside of the neighbourhood of the singular point during the iterative process which subsequently become smaller and disappear. The following technique is used in order that the expression $2Y(\psi^{(n)} + \psi_\theta^{(n)})\sin\theta$ should not vanish (and $T^{(n)}$, $U^{(n)}$ become infinite) and that the iterative process should not diverge. If in $\Sigma_1' = \{(\xi_i, \eta_j) \in \Sigma_1\}$ $\min[Y(\psi^{(n)}) + 0.5\psi_\theta^{(n)}\sin\theta] = -m^{(n)} < 0$, then we put $Y^{(n)} = Y(\psi^{(n)}) + 3m^{(n)}\{1-\exp[\delta_1(\xi-\xi_a)^2 + \delta_2\eta^2]\}$, and, otherwise we put $Y^{(n)} = Y(\psi^{(n)})$.

The function $\psi^{01} + \psi^{02}$ generates a broad range of negative values of $Y(\psi^{01} + \psi^{02})$. Allowing for this, it is convenient to put $\psi^0 = \psi^{01} + \psi^{02}\exp[\delta_3(\tau-1)^2 + \delta_4\theta^2]$, where $\delta_3,\ \delta_4 \in [-20,\ -10]$. A function ψ^0 constructed using the technique described above retains all the required properties, which are inherent in the sum $\psi^{01} + \psi^{02}$, in the neighbourhood of the singular point and, at the same time, it is such that $Y(\psi^0) > 0$ in Σ_0.

The use of formula (1.12) is not the only possible method for determining $Y(\psi^{(n)})$. Moreover, this method does suffer from the drawback that the maximum errors which arise in determining $\chi^{(n)}$ in the neighbourhood of a singular point, as a result of integration, extend upwards from and to the right of this neighbourhood. Formula (1.2) is therefore used to determine $Y(\psi^{(n)})$ only at the beginning of the iterative process. Later, $Y(\psi^{(n)})$ is determined using alternate integration of relationships (1.8) along those trajectories which pass around the neighbourhood of the singular point or just terminate at it. here, a spline approximation of the partial derivatives of $\chi^{(n)}$ occurring in expressions (1.8) is used.

It can be shown that the condition $G_\theta = H_\tau$, which follows from (1.8), is equivalent to Eq. (1.10). When this condition is satisfied, the value of y, found at an arbitrary point of the domain Σ_0 by integration of the expression $dy = Gd\tau + Hd\theta$, is independent of the integration path. This consideration can be made use of in controlling the accuracy of the solution obtained.

The proposed method of calculation can be used to investigate compressible liquid and gas flows with different dependences $M(\lambda)$, $\nu(\lambda)$. An experiment we carried out suggests that, in the case of the relations $M(\lambda)$, $\nu(\lambda)$, which hold in the case of an ideal gas, there is a fairly wide range of governing parameters for which the process of successive approximations, which has been described above, converges in a stable manner.

Having determined $\psi(\tau,\ \theta)$, $r(\tau,\ \theta)$ simultaneously using the method described, there is no difficulty in finding all the required characteristics of the flow using (1.7)

4. We know [14] that, for many liquids, the relation between the pressure p and the density ρ in an isentropic process is given by the formula

$$\frac{p+B}{p_s+B}=\left(\frac{\rho}{\rho_s}\right)^k \tag{4.1}$$

where p_s, ρ_s, B, k are certain constants. For water $k=7.15$ [15]. The relationships

$$M^2=\frac{2}{k+1}\lambda^2(1-m\lambda^2)^{-1},\quad \rho=\rho_0(1-m\lambda^2)^{1/(k-1)} \tag{4.2}$$

$$p=p_0+\frac{k+1}{2k}\rho_0 a_*^2[(1-m\lambda^2)^{k/(k-1)}-1],\quad m=\frac{k-1}{k+1} \tag{4.3}$$

which are used below (a_* is the critical velocity and p_0 is the value of p at the stagnation point) are obtained from Bernoulli's equation and (4.1) subject to the assumptions which have been made.

Let p_a, ρ_a, V_a be the pressure, density and velocity of the free flow, p_c the pressure in the cavern, X the cavitator resistance and R_0 the radius of the cavitator (the value of r at the point C). Moreover, $R_0=1$. Using (4.2) and (4.3), we obtain the expressions

$$Q=\frac{k+1}{k\lambda_a^2}(1-m\lambda_a^2)^{1/(1-k)}[(1-m\lambda_a^2)^{k/(k-1)}-(1-m\lambda_c^2)^{k/(k-1)}] \tag{4.4}$$

$$C_x=\frac{2(1-m\lambda_a^2)^{1/(1-k)}}{r^2(\tau_0,\theta_0)}\int_0^{\tau_0} r^2(\tau,\theta_0)(1-m\lambda_a^2\tau^2)^{1/(k-1)}\tau d\tau \tag{4.5}$$

for the cavitation number $Q=2(p_a-p_c)/(\rho_a V_a^2)$ and the drag coefficient $C_x=2X/(\pi\rho_a V_a^2 R_0^2)$. The equality $Q=\tau_0^2-1$. is obtained from (4.4) in the limit when λ_a, $\lambda_c\to 0$. When $\theta_0=\pi$, we have $r(\tau,\ \theta_0)=r(\tau_0,\ \theta_0)$ and, in this case, it therefore follows from (4.5) that

$$C_x=\frac{k+1}{k\lambda_a^2}(1-m\lambda_a^2)^{1/(1-k)}[1-(1-m\lambda_c^2)^{k/(k-1)}]\quad (\theta_0=\pi) \tag{4.6}$$

$C_x=\tau_2^0=1+Q$ is obtained from (4.6) in the limit subject to the additional condition λ_a, $\lambda_c\to 0$.

Let us consider a cavitator where the part around which the flow occurs is in the form of a cone with an aperture angle θ_0 in the range $[\pi/2,\pi]$ (a disc or cavitator with a conical channel). Let E and F be points on the arc CDG of the free surface at which $\theta=\pi/2$ and $\theta=-\pi/2$ respectively. Let R_1 and R_2 be the values of r at points E and D (R_2 is the radius of the cavern in the plane of symmetry), let L_0, L_1 and L_2 be the projections of the vectors BH, CG and EF on to the x-axis (L_2 is the length of the cavern, $L_1=L_0-2R_0\mathrm{ctg}\theta_0$), and let K_1 be the curvature of the arc CDG at point D.

One version of the calculation using the proposed model assumes that the values of k, θ_0, Q, λ_c are specified. However, for the case when $\lambda_c=0$ (an incompressible fluid), the solution is independent of k. Furthermore, when $\lambda_c=0$, the solution is general for all fluid models.

In Table 1, we present the values of C_x, L_2 and R_2, obtained for a series of values of Q in the calculation of the cavitational flow around a disc ($\theta_0=\pi/2$) by an incompressible fluid using the method which has been described on an $I\times J=50\times 50$ grid and also the corresponding values from [4]. A comparison shows that the differences are less than one percent. A similar estimate may be obtained by comparing the data presented below with the values of C_x, L_2 R_2 found using Guzevskii's approximation formulae [3].

5. Calculations were carried out on an $I\times J=50\times 100$ grid for flows of an incompressible fluid around cavitators with aperture angles of the cones $\theta_0=\pi(6+n)/12$ ($n=0,1,\ldots,6$) for $Q=0.15$; 0.2; 0.3; 0.4; 0.5. Approximation formulae for the parameter C_x, K_1, R_1, R_2, L_1, L_2 of the following form were constructed on the basis of the results obtained

Table 1

Q	R_2	R_2 [4]	L_2	L_2 [4]	C_x	C_x [4]
0.2636	2.1184	2.115	10.043	10	1.0539	1.0513
0.1477	2.6773	2.678	20.121	20	0.9559	0.9516
0.1048	3.1045	3.109	30.105	30	0.9198	0.9150
0.0819	3.4607	3.471	40.176	40	0.9021	0.8955
0.0676	3.7693	3.788	50.257	50	0.8917	0.8833

$$
\begin{aligned}
&C_x^0 = a + b(1+Q)^{-1} + cQ, && d = d_1 + d_2 \ln\omega_1 + d_3 \ln^2\omega_1 \\
&K_1^0 = aQ + bQ^2 + cQ^3, && d = d_1 + d_2 \ln\omega_2 + d_3\omega_2^2 \\
&R_1^0 = 1 + a + bQ^{-1}\ln Q + cQ^{-1}, && d = d_1\omega_3 + d_2\omega_3^2 + d_3\omega_3^3 \\
&P^0 = a + bQ^{-1}\ln Q + cQ^{-1}, && d = d_1 + d_2\ln\omega_2 + d_3\omega_2^2, \quad P^0 = R_2^0, L_1^0, L_2^0 \\
&\omega_1 = \theta_0 + \frac{\pi}{12}, \quad \omega_2 = \theta_0 + \frac{\pi}{2}, && \omega_3 = \theta_0 - \frac{\pi}{2}; \quad d = a, b, c
\end{aligned}
\tag{5.1}
$$

(values of the parameters in the case when $\lambda_c = 0$ are given the superscript 0).

The values of the coefficients a_j, b_j and c_j for the approximation formulae and the values ε of the maximum relative errors of the approximation are shown in Table 2. The differential characteristic K_1 is calculated with a lower accuracy than the remaining integral and local characteristics, and the value of ε for K_1 is therefore substantially greater than for the other parameters.

Table 2

	d_j	$j = 1$	2	3	$10^4 \times \varepsilon$
	a_j	0.08016	1.35824	–0.49618	
C_x^0	b_j	0.34607	–0.48921	0.16908	5
	c_j	0.80092	0.26089	–0.07998	
	a_j	0.04769	–0.01338	0.00018	
K_1^0	b_j	1.08518	–0.48366	0.01198	137
	c_j	–0.38103	0.18322	–0.00483	
	a_j	0.01638	0.01924	–0.01035	
R_1^0	b_j	0.00356	0.00025	–0.00058	13
	c_j	0.00818	0.00063	–0.00142	
	a_j	0.61572	0.37494	–0.00857	
R_2^0	b_j	–0.05025	0.14946	–0.00319	16
	c_j	–0.05503	0.51617	–0.01131	
	a_j	0.53405	–0.32871	0.00528	
L_1^0	b_j	–0.57861	–0.00703	–0.00194	22
	c_j	–0.19199	2.18048	–0.05274	
	a_j	0.61528	–0.46052	0.01232	
L_2^0	b_j	–0.57545	–0.01431	–0.00142	20
	c_j	–0.18529	2.16486	–0.05160	

The following expression is obtained from (5.1) for the drag coefficient of the disc $(\theta=\pi/2)$ in an incompressible fluid

$$C_x^0=0.7208+0.1118(1+Q)^{-1}+0.9296Q \quad (0.15\leqslant Q\leqslant 1) \tag{5.2}$$

It is interesting to compare (5.2) with the approximation formulae

$$C_x^0=(0.827+0.026Q)(1+Q) \quad (0.1\leqslant Q\leqslant 0.6) \tag{5.3}$$

$$C_x^0=0.82825+0.86Q \quad (0\leqslant Q\leqslant 0.25) \tag{5.4}$$

of which the first, which is given in [16, p. 458] was constructed on the basis of a graph from [17] and the second is obtained, when $\theta_0=\pi/2$ from Guzevskii's formula for a cone [3]. The differences in the values of C_x^0, calculated using formulae (5.2) and (5.3) in the interval $0.1\leqslant Q\leqslant 1$ do not exceed 0.3% and 0.075% in the interval $0.25\leqslant Q\leqslant 1$. The difference in the values of C_x^0, calculated using formulae (5.2) and (5.4) do not exceed 0.15% in the range $0.1\leqslant Q\leqslant 0.5$ and 0.06% in the range $0.15\leqslant Q\leqslant 0.4$.
The nature of the approximation relations (5.1) can be described by the inequalities

$$\frac{\partial L_1^0}{\partial Q}<0,\ \frac{\partial L_2^0}{\partial Q}<0,\ \frac{\partial R_2^0}{\partial Q}<0,\ \frac{\partial K_1^0}{\partial Q}>0,\ \frac{\partial C_x^0}{\partial Q}>0,\ \frac{\partial C_x^0}{\partial \theta_0}>0 \tag{5.5}$$

$$\frac{\partial S^0}{\partial \theta_0}\geqslant 0, \quad S^0=-K_1^0,\ R_1^0,\ R_2^0,\ L_1^0,\ L_2^0 \tag{5.6}$$

where the inequalities (5.5) hold over the whole computational domain $0.15\leqslant Q\leqslant 1$, $\pi/2\leqslant\theta_0\leqslant\pi$ while (5.6) are satisfied outside a certain neighbourhood of the range $0.15\leqslant Q\leqslant 1$, $\theta_0=\pi$, where the values of $|\partial S^0/\partial\theta_0|$ are small compared with the corresponding values when $\theta_0=\pi/2$.
The inequalities

$$\frac{\partial(L_2^0/L_1^0)}{\partial\theta_0}>0,\quad \frac{\partial(L_2^0/L_1^0)}{\partial Q}>0,\quad \frac{\partial(R_2^0/L_2^0)}{\partial Q}>0$$

also hold for the whole of the computational domain.

The quantities R_2^0/L_2^0 and $C_{x1}^0=C_x^0/(R_2^0)^2$ (C_{x1} is the drag coefficient determined over the midsectional area of the cavern) depend weakly on θ_0 and the quantity R_2^0/L_2^0 attains its maximum values when $\theta_0=\pi/2$. One unexpected fact is that the quantity R_0, which characterizes the position of the points for which $\theta=\pm\pi/2$, does not take its maximum values on the boundary of the computational domain (when $Q=0.15$) but when $Q=0.3$ for all values of θ_0 in the range $(\pi/2, \pi)$.

The dependence of some of the parameters on θ_0 when $\lambda_c=0$ and $Q=0.15$ is shown in Fig. 3. Curves 1–4 correspond to the quantities $0.1\times L_2^0$, C_x^0+1, $100\times K_1^0$, R_2^0-1. Curve 5 shows how C_x^0+1 depends on θ_0; it was obtained on the assumption that the pressure distribution along the generatrix of the cone can be found from the solution of the corresponding planar problem, as was done in [18, 19].

In Fig. 4, curves 1–3 depict the dependence $L_0^0(\theta_0)$ for $Q=0.15$, 0.3 and 1, respectively. For each Q, a $\omega_0=\omega_0(Q)$ exists such that $L_0^0=0$ when $\theta_0=\omega_0$ $(d\omega_0/dQ<0)$, $L_0^0<0$ when $w_0<\theta_0\leqslant\pi$, $L_0^0\to-\infty$ when $\theta_0\to\pi$. The domain occupied by the flow in the (x,r)-plane is therefore two-sheeted when $\omega_0<\theta_0\leqslant\pi$. A diagram of the flow when $\omega_0<\theta_0\leqslant\pi$ and $\theta_0=\pi$ is shown in Figs 5(a) and (b).

6. Calculations were carried out on the cavitational flow around a disc $(\theta_0=\pi/2)$ by a stream of water $(k=7.15)$ for $Q=0.15; 0.2; 0.3; 0.4; 0.5$; $M_c^2=0.2; 0.4; 0.6; 0.8; 1$ (M_c is the value of the Mach number when $\lambda=\lambda_c$) on an $I\times J=50\times 50$ grid. Approximation formulae were constructed for the parameters C_x, K_1, L_2, R_2 ($L_0=L_1=L_2$, $R_1=1$ when $\theta_2=\pi/2$) on the basis of the results obtained. These have the following form

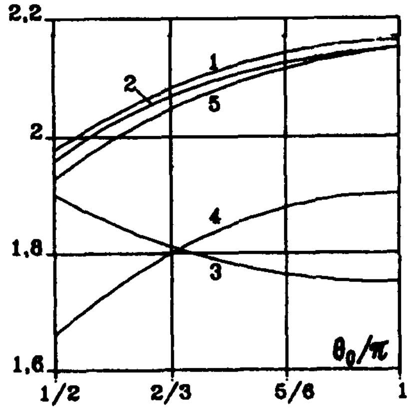

Fig. 3.

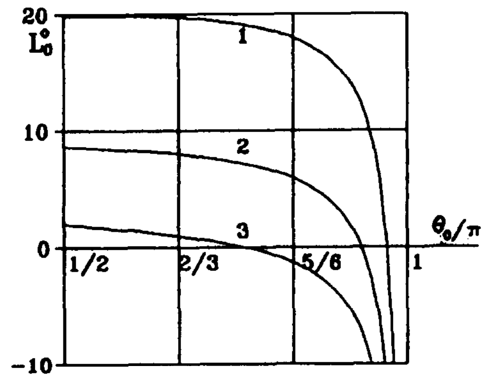

Fig. 4.

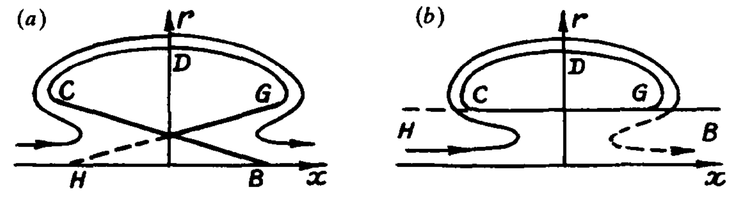

Fig. 5.

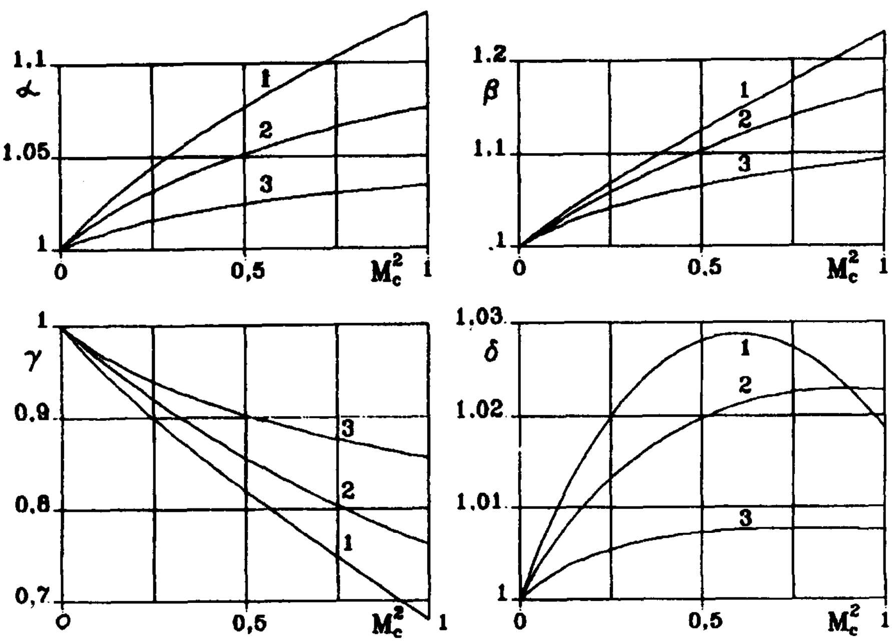

Fig. 6.

$$C_x=\alpha C_x^0,\quad L_2=\beta L_2^0,\quad K_1=\gamma K_1^0,\quad R_2=\delta R_2^0 \tag{6.1}$$

$$\mu=1+fM_c^2+gM_c^3+hM_c^4,\quad \mu=\alpha,\beta,\gamma,\delta,\qquad d=d_1+d_2Q^{-1}\ln Q+d_3Q^{-1},\quad d=f,g,h$$

Here, C_x^0, L_2^0, K_1^0, R_2^0 are functions of Q obtained in accordance with formulae (5.1) and Table 2 when $\theta_0=\pi/2$. The values of the coefficients f_j, g_j, h_j for the approximation formulae (6.1) and of ε, the maximum relative errors in the approximation are presented in Table 3.

Table 3

	d_j	$j=1$	2	3	$10^4\times\varepsilon$
	f_j	−0.00599	0.03688	0.10945	
α	g_j	−0.01706	−0.02887	−0.08055	9
	h_j	0.01808	0.00201	0.00992	
	f_j	0.29589	−0.01529	−0.02098	
β	g_j	−0.57432	0.13310	0.30976	23
	h_j	0.28978	−0.09167	−0.20660	
	f_j	−0.50861	0.05969	0.10518	
γ	g_j	0.89609	−0.23137	−0.51375	127
	h_j	−0.42967	0.14014	0.30676	
	f_j	−0.00028	0.01514	0.04757	
δ	g_j	−0.02871	−0.01376	−0.03329	11
	h_j	0.01245	0.00858	0.00988	

The nature of the dependence of the quantities α, β and γ on Q and M_c in the domain $Q\in[0.15, 1]$, $M_c\in[0, 1]$ can be described by the inequalities

$$\frac{\partial\alpha}{\partial M_c}>0,\quad \frac{\partial\alpha}{\partial Q}<0,\quad \frac{\partial\beta}{\partial M_c}>0,\quad \frac{\partial\beta}{\partial Q}<0,\quad \frac{\partial\gamma}{\partial M_c}<0,\quad \frac{\partial\gamma}{\partial Q}>0$$

Hence, as M_c increases for a fixed value of $Q\in[0.15, 1]$, C_x and L_2 become larger while K_1 decreases and, moreover, the change in the parameters is greater, the smaller the value of Q.

The parameters α, β, γ and δ as functions of M_c^2 when $Q=0.15$, 0.35 and 1 are shown in Fig. 6 (curves 1–3 respectively). The relation $\delta(M_c, Q)$ is quite complex. As Q decreases, the value of M_c at which $\delta(M_c, Q)$ attains its maximum value δ_m decreases, but δ_m itself increases. When Q is reduced, the magnitude of $\delta(1, Q)$ increases when $Q\in[1, 1]$, where $q=0.235$ and falls off when $Q\in[0.15, q]$.

The greatest change in the parameters α, β, γ and δ (the greatest change in the quantities C_x, L_2, K_1 and R_2 because of the compressibility of water in the case of a disc) was 12.7, 22.9, 32.2 and 2.9%, respectively, in the range $Q\in[0.15, 1]$, $M_c\in[0, 1]$

Putting $L=L_2/(2R_2)$, $L^0=L_2^0/(2R_2^0)$, $\sigma=\beta/\delta$, we write by analogy with (6.1), the equality $L=\sigma L^0$ (L is the length of the cavern, $\sigma=\sigma(M_c, Q)$). It follows from the approximation relationships (6.1) that the inequalities $\partial\sigma/\partial M_c>0$, $\partial\sigma/\partial Q<0$ hold in the range $Q\in[0.15, 1]$, $M_c\in[0, 1]$. According to the last of these inequalities, the effect of compressibility on the elongation of the cavern increases as Q decreases. Meanwhile, an analysis of the solution of the problem in a linearized formulation leads to a contradictory assertion [9, 10]. This apparent contradiction is explained by the difference in the domains of applicability of the results: the results obtained in [9, 10], using the assumptions of the gas dynamics of a thin body, are valid for small Q numbers (the smaller the value of Q, the more accurate are the equations) and for M_c numbers which are sufficiently far from unity.

The dependence of the elongation L of the cavern behind the disc on the number Q for the values $M_c=0$, 0.6 and 1 is shown in Fig. 7 (curves 1–3 respectively). In order to construct curves 1 and 2 in the interval $0.075\leqslant Q\leqslant 0.15$, the left boundary of which is marked by means of a broken line, the results of the solution of four additional versions of the problem were used. These versions correspond to the values $Q=0.075$, 0.1 and $M_c=0$, 0.6 (it is not possible to obtain a solution for these values of Q when $M_c=1$). As Q decreases in the range $0.075\leqslant Q\leqslant 0.1$, the ratio of the ordinates of curves 2 and 1, which is equal to $\sigma(0.6, Q)$, approaches unity. This confirms the non-contradictory nature of the results of the linear and non-linear theories. At the same time, there are no reasons to expect that the ratio of the ordinate of curves 1 and 3 will tend to unity when $Q\to 0$ when they are extended into the region of small Q values.

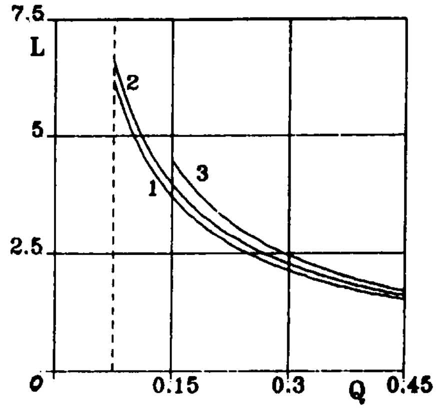

Fig. 7.

7. Further calculations were carried out on 36 versions of the governing parameters corresponding to the values $k = 7.15; Q = 0.15; 0.2; 0.4; 1; M_c^2 = 0.5; 0.75; 1; \theta_0 = 2\pi/3,\ 5\pi/6,\ \pi$. It was established that, in the calculation of the parameters L_1, L_2, K_1, R_2 using the approximation formulae

$$L_1 = \beta L_1^0, \quad L_2 = \beta L_2^0, \quad K_1 = \gamma K_1^0, \quad R_2 = \delta R_2^0 \tag{7.1}$$

where L_1^0, L_2^0, K_1^0, R_2^0 are functions of Q and θ_0 defined by formulae (5.1) and Table 2 and β, γ and δ are functions of Q and M_c defined by formulae (6.1) and Table 3, the resulting error does not exceed 2% for K_1 and 1.5% for L_1, L_2 and R_2. Formulae (7.1) are therefore suitable for use in the range $Q \in [0.15, 1]$, $M_c \in [0, 1]$, $\theta_0 \in [\pi/2, \pi]$.

To calculate C_x over the same range, it is better not to use a formula of the form (7.1) but the approximation relationship

$$C_x = C_x' + 4(C_x'' - C_x')(1 - \theta_0/\pi)^2$$

where C_x' is the exact value of C_x when $\theta_0 = \pi$ (see (4.6)) and C_x'' is the value of C_x when $\theta_o = \pi/2$ calculated using the first of formulae (6.1). The resulting error in this case does not exceed 0.5%.

It is not possible to propose a convenient approximation formula for calculating the values of R_1 when $M_c \neq 0$. However, it may be stated that, as M_c increases from zero to unity, the magnitude of R_1 decreases, but by not more than 1%.

It follows from the determination of the cavitation number Q that

$$M_a^2 = 2(p_a - p_v)/(\rho_a a_a^2 Q) \tag{7.2}$$

holds in the case of steam cavitation, where p_v is the pressure of the saturated vapours at the specified temperature and M_a' and a_a are the values of the number M and the velocity of sound a in the unperturbed flow. By using the well-known values of p_v, ρ_a, a_a c and (7.2), it can be readily shown that, under normal conditions when the pressure of the approach stream p_a is of the order of 1 atmosphere, negligibly small values of M_a and M_c correspond to the value $Q = 0.15$. As p_a increases at the fixed value of $Q = 0.15$, the values of M_a and M_c become larger but it is only at a value of p_a of the order of 900 atmospheres that M_c attains a value of unity. From what has been said, it has to be acknowledged that (6.1) and (7.1) are of greater theoretical than practical interest.

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